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Emmanuel Fontaine and Raymond Cointe

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# A slender body approach to nonlinear bow waves

By Emmanuel Fontaine[1] and Raymond Cointe[2]
[1] *Bassin d'Essais des Carènes, 27100 Val de Reuil, France*
[2] *Ministère de l'Environnement, 20 avenue de Ségur, 75302 Paris 07 SP, France*

The behaviour of the flow near the bow of a slender ship is studied. The fluid is assumed to be perfect and incompressible and the flow to be irrotational. The formalism of matched asymptotic expansion is used to provide a consistent perturbation procedure for the simplification of the initial problem. The resulting nonlinear free surface problem describing the flow in the inner domain close to the bow is solved numerically. Examples of solutions are given for the flows around a wedge shaped bow and a prismatic planing hull.

## 1. Introduction

The potential free surface flow around a slender ship has led to many studies based on a 'parabolic' approximation of the equations. This approach, initially introduced by Ogilvie (1967), consists in reducing the three-dimensional nonlinear stationary problem to a set of two-dimensional transient problems, which can, in some circumstances, be linearized. Ogilvie (1972) solved analytically the linear problem and obtained a simplified solution for the wave profile around a wedge shaped bow. Chapman (1975) used a similar theory to study the flow around a flat plate oscillating in yaw and sway. More recently, this theory was used by Faltinsen & Zhao (1991) to study the flow around a high speed ship and by Maruo & Song (1994) to study the bow flow and deck wetness of a ship in waves.

In this paper, we use the method of matched asymptotic expansions to provide a consistent perturbation procedure for the justification of the 'parabolic' approximation for the determination of the free surface flow around a slender ship. This suggests that this theory can apply either to the flow around a slender ship at high Froude number based on the length or to the flow near the bow of a slender ship at high Froude number based on the draught. We show some numerical computations based on this approximation and using a mixed Eulerian–Lagrangian method to solve the nonlinear inner problem. We present comparisons with experimental results and other asymptotic theories.

## 2. The bow flow: inner problem

### (*a*) *Assumptions and definitions*

The fluid is assumed to be incompressible and the flow irrotational. The ship is defined by its maximum beam $b$ and draught $h$, see figure 1. The shape of the hull

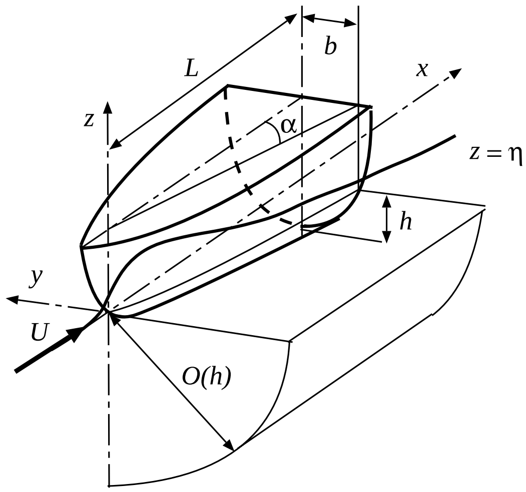


Figure 1. Geometrical definitions.

is given by $y/b = f(x/L, z/h)$ where $L$ is the longitudinal length scale. The two non-dimensional parameters describing the shape of the hull are $\tan\alpha = b/L$ and $\delta = h/L$. The ship will be said to be *slender* when $\tan\alpha \simeq \delta \ll 1$, to be *thin* when $\tan\alpha \ll \delta \ll 1$, and to be *flat* when $\delta \ll \tan\alpha \ll 1$.

### (*b*) *Simplified equations for a slender ship*

To study the flow in the vicinity of the ship, an inner domain is defined on a radial length scale equal to the draught $h$ and on the longitudinal length scale $L$. The inner variables are defined as

$$\tilde{x} = \frac{x}{L}, \quad \hat{y} = \frac{y}{h}, \quad \hat{z} = \frac{z}{h}, \quad \hat{\varphi} = \frac{\varphi}{Ub}, \quad \hat{\eta} = \frac{\eta}{b}, \tag{2.1}$$

where $\varphi$ is the velocity potential and $\eta$ the free surface elevation. Assuming the ship to be slender, the following asymptotic expansions are introduced:

$$\hat{\varphi}(\tilde{x}, \hat{y}, \hat{z}; \alpha, \delta) = \hat{\mu}_1(\alpha, \delta)\, \hat{\varphi}_1(\tilde{x}, \hat{y}, \hat{z}) + o(\hat{\mu}_1), \tag{2.2}$$

$$\hat{\eta}(\tilde{x}, \hat{y}; \alpha, \delta) = \hat{\nu}_1(\alpha, \delta)\, \hat{\eta}_1(\tilde{x}, \hat{y}) + o(\hat{\nu}_1). \tag{2.3}$$

The principle of least degeneracy allows to determine the gauges $\hat{\mu}_1$ and $\hat{\nu}_1$, leading to $\hat{\mu}_1 = \delta$ and $\hat{\nu}_1 = 1$. A non-trivial solution for $\hat{\eta}_1$ can then only be found if

$$\hat{F}_L^2 = \frac{\delta U^2}{gL} \geqslant O(1). \tag{2.4}$$

The leading order perturbation potential $\hat{\varphi}_1$ satisfies the two-dimensional Laplace equation in the fluid domain of each transverse plane:

$$\frac{\partial^2 \hat{\varphi}_1}{\partial \hat{y}^2} + \frac{\partial^2 \hat{\varphi}_1}{\partial \hat{z}^2} = 0, \tag{2.5}$$

with the following boundary conditions on the hull $\hat{y} = (\tan\alpha/\delta) f(\tilde{x}, \hat{z})$:

$$\frac{\partial f}{\partial \tilde{x}} - \frac{\partial \hat{\varphi}_1}{\partial \hat{y}} + \frac{\tan\alpha}{\delta} \frac{\partial f}{\partial \hat{z}} \frac{\partial \hat{\varphi}_1}{\partial \hat{z}} = 0, \tag{2.6}$$

and on the free surface $\hat{z} = (\tan\alpha/\delta)\hat{\eta}_1$:

$$\frac{\partial\hat{\eta}_1}{\partial\tilde{x}} + \frac{\tan\alpha}{\delta}\frac{\partial\hat{\varphi}_1}{\partial\hat{y}}\cdot\frac{\partial\hat{\eta}_1}{\partial\hat{y}} - \frac{\partial\hat{\varphi}_1}{\partial\hat{z}} = 0 \tag{2.7}$$

$$\frac{\partial\hat{\varphi}_1}{\partial\tilde{x}} + \frac{1}{2}\frac{\tan\alpha}{\delta}\left(\left(\frac{\partial\hat{\varphi}_1}{\partial\hat{y}}\right)^2 + \left(\frac{\partial\hat{\varphi}_1}{\partial\hat{z}}\right)^2\right) + \frac{\hat{\eta}_1}{\hat{F}_L^2} = 0 \tag{2.8}$$

These equations describe the first order inner problem for a slender ship. They also apply when the ship is assumed to be thin, but in this last case the boundary conditions can moreover be linearized.

### (*c*) *Asymptotic domain of validity*

Apart from the slender ship hypothesis, the main assumption leading to the asymptotic expansion is given by equation (2.4). If $L$ is the length of the ship, this equation implies that $U$ must satisfy

$$F_L = \frac{U}{\sqrt{gL}} \geqslant \frac{1}{\sqrt{\delta}} = \sqrt{\frac{L}{h}}. \tag{2.9}$$

This confirms that these asymptotic equations can be applied to study the flow around a slender high speed ship (Chapman 1975; Faltinsen & Zhao 1991). When the Froude number $F_L$ is much larger than $1/\sqrt{\delta}$, the effects of gravity can moreover be neglected. This case corresponds in practice to the flow around a planing hull (e.g. Tulin 1956).

Even for a slower ship, this approximation also applies at a distance $L$ from the bow if $L$ is chosen so that

$$L \leqslant U\sqrt{\frac{h}{g}} = hF_h, \quad F_h = \frac{U}{\sqrt{gh}}. \tag{2.10}$$

This equation gives the order of magnitude of $L$ for the approximation to remain valid as long as the ship is assumed to be slender or thin. These hypotheses require that

$$F_h \gg 1. \tag{2.11}$$

This asymptotic theory can therefore also be applied to study the flow around the bow of a slender ship if the draught Froude number is much greater than one.

## 3. The far field flow: outer problem

The inner solution only satisfies the two-dimensionnal Laplace equation. It cannot therefore be valid far from the ship. Boundary conditions at infinity for the inner problem must be provided by a matching condition with an outer solution.

The outer domain is defined on the length scale $L$:

$$\tilde{x} = \frac{x}{L}, \quad \tilde{y} = \frac{y}{L} = \delta\,\hat{y}, \quad \tilde{z} = \frac{z}{L} = \delta\hat{z}, \quad \tilde{\eta} = \hat{\eta}, \quad \tilde{\varphi} = \hat{\varphi}, \tag{3.1}$$

and the following asymptotic expansion is performed:

$$\tilde{\varphi}(\tilde{x},\tilde{y},\tilde{z};\alpha,\delta) = \tilde{\mu}_1(\alpha,\delta)\,\tilde{\varphi}_1(\tilde{x},\tilde{y},\tilde{z}) + o(\tilde{\mu}_1), \tag{3.2}$$

$$\tilde{\eta}(\tilde{x},\tilde{y};\alpha,\delta) = \tilde{\nu}_1(\alpha,\delta)\,\hat{\eta}_1(\tilde{x},\tilde{y}) + o(\tilde{\nu}_1). \tag{3.3}$$

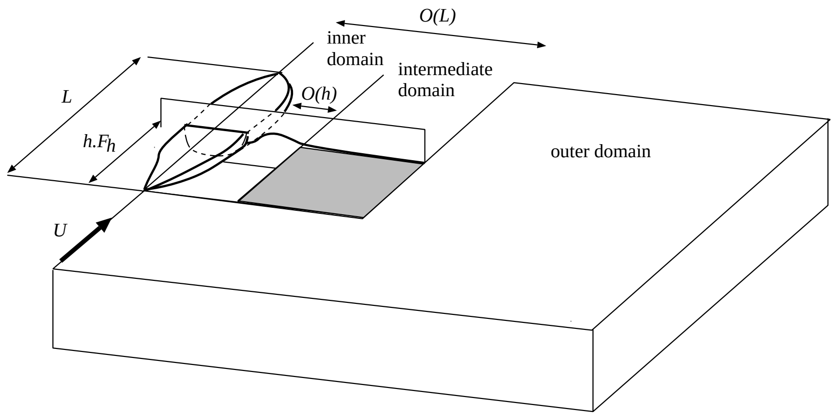


Figure 2. Matching procedure.

The outer domain equations are obtained using the principle of least degeneracy, leading to $\tilde{\mu}_1 = \tilde{\nu}_1$, the three-dimensional Laplace equation in the fluid domain and the free surface boundary conditions

$$\frac{\partial \tilde{\varphi}_1}{\partial \tilde{x}}(\tilde{x}, \tilde{y}, 0) = 0, \quad \left[\frac{\partial \tilde{\eta}_1}{\partial \tilde{x}} - \frac{\partial \tilde{\varphi}_1}{\partial \tilde{z}}\right](\ell \tilde{x}, \tilde{y}, 0) = 0. \tag{3.4}$$

The general solution of this problem can be found as a multipole expansion (Ward 1955):

$$\tilde{\varphi}_1(\tilde{x}, \tilde{r}, \theta) = \sum_{n=1}^{\infty} \frac{\sin(n\theta)}{\tilde{r}^n} \int_0^{\infty} \frac{\left[\sqrt{(\tilde{x}-s)^2 + \tilde{r}^2} + \tilde{x} - s\right]^n}{\sqrt{(\tilde{x}-s)^2 + \tilde{r}^2}} f_n(s)\, \mathrm{d}s. \tag{3.5}$$

As $\tilde{r}$ goes to zero, this leads to the following expansion (for $\tilde{x} \neq 0$):

$$\lim_{\tilde{r} \to 0} \tilde{\varphi}_1(\tilde{x}, \tilde{r}, \theta) = \sum_{n=1}^{\infty} \frac{2^n \sin(n\theta)}{\tilde{r}^n} \int_0^{\tilde{x}} (\tilde{x} - s)^{n-1} f_n(s)\, \mathrm{d}s (1 + o(1)). \tag{3.6}$$

This asymptotic expansion is uniformly valid for all negative values of $\tilde{x}$. On the body, the outer solution is singular and must be matched to the inner solution. The unknown multipole intensity $f_n(\tilde{x})$ and order of magnitude $\tilde{\mu}_1$, $\tilde{\nu}_1$ of the outer solution are thus to be determined from the matching.

## 4. Matching

### (a) *The intermediate domain*

The inner problem is nonlinear and cannot be solved analytically. It is therefore not possible to give an analytical description of the behaviour at infinity of the inner solution. In order to match the inner and outer solutions, an intermediate domain (Kaplun & Lagerstrom 1957) is introduced. The intermediate solution must match the inner solution at the origin and the outer solution at infinity.

The intermediate variables are defined as

$$\bar{r} = \frac{\tilde{r}}{\zeta(\delta)}, \quad \bar{y} = \frac{\tilde{y}}{\zeta(\delta)}, \quad \bar{z} = \frac{\tilde{z}}{\zeta(\delta)}, \quad \bar{\varphi} = \tilde{\varphi}, \quad \bar{\eta} = \tilde{\eta}, \tag{4.1}$$

with $\delta \ll \zeta(\delta) \ll 1$ and the following asymptotic expansion is assumed:

$$\bar{\varphi}(\tilde{x},\bar{r},\theta;\alpha,\delta) = \bar{\mu}_1(\alpha,\delta)\,\bar{\varphi}_1(\tilde{x},\bar{r},\theta) + o(\bar{\mu}_1), \tag{4.2}$$

$$\bar{\eta}(\tilde{x},\bar{y};\alpha,\delta) = \bar{\nu}_1(\alpha,\delta)\,\bar{\eta}_1(\tilde{x},\bar{y}) + o(\bar{\nu}_1). \tag{4.3}$$

The principle of least degeneracy implies $\bar{\mu}_1 = \zeta\,\bar{\nu}_1$. Because of the dilatation of the transverse coordinates in the intermediate domain ($\zeta \ll 1$), $\bar{\varphi}_1$ satisfies the two-dimensional Laplace equation with the free surface conditions

$$\bar{\varphi}_1(\tilde{x},\bar{y},0) = 0, \qquad \left[\frac{\partial\bar{\eta}_1}{\partial\tilde{x}} - \frac{\partial\bar{\varphi}_1}{\partial\bar{z}}\right](\tilde{x},\bar{y},0) = 0. \tag{4.4}$$

Thus $\bar{\varphi}_1$ satisfies the simpler aspects of each of the two asymptotic solutions, i.e. the two-dimensional Laplace equation of the inner problem and the linear boundary conditions of the outer problem. The general solution of this problem can be expressed in term of a Laurent expansion:

$$\bar{\varphi}_1(\tilde{x},\bar{r},\theta) = \sum_{n=1}^{\infty}\sin(n\theta)\left[\frac{A_n(\tilde{x})}{\bar{r}^n} + B_n(\tilde{x})\bar{r}^n\right]. \tag{4.5}$$

### (b) Matching the various solutions

Matching the intermediate to outer solutions leads one to retain only the first term ($n = 1$) of the two multipole expansions (4.5) and (3.5). Thus, the outer perturbation potential is represented by a vertical three-dimensional dipole distribution on the $\tilde{x}$ axis. In order to satisfy the outer free surface condition, the dipole intensity $\mu$ must be constant, equal to $\mu_{\mathrm{w}}$ in the wake behind the the ship. Then,

$$\tilde{\varphi}_1 = \int_0^1 -\frac{\mu(s)}{4\pi}\frac{\tilde{r}\sin(\theta)}{[(\tilde{x}-s)^2+\tilde{r}^2]^{\frac{3}{2}}}\,\mathrm{d}s - \frac{\mu_{\mathrm{w}}}{4\pi}\frac{\sin(\theta)}{\tilde{r}}\left(1+\frac{\tilde{x}-1}{\sqrt{(\tilde{x}-1)^2+\tilde{r}^2}}\right), \tag{4.6}$$

$$\tilde{\eta}_1 = \frac{1}{\tilde{y}^2}\int_0^1 -\frac{\mu(s)}{4\pi}\left[1+\frac{\tilde{x}-s}{\sqrt{(\tilde{x}-s)^2+\tilde{y}^2}}\right]\mathrm{d}s - \frac{\mu_{\mathrm{w}}}{4\pi\tilde{y}^2}\left[\tilde{x}-1+\sqrt{(\tilde{x}-1)^2+\tilde{y}^2}\right]. \tag{4.7}$$

Matching the intermediate and the inner solutions gives the order of magnitude of the outer solution as $\tilde{\mu}_1 = \tilde{\nu}_1 = \delta^2$ and the behaviour of the inner solution at infinity:

$$\lim_{\hat{r}\to\infty}\hat{\varphi}_1(\tilde{x},\hat{r},\theta) = -\frac{\mu(\tilde{x})}{2\pi}\cdot\frac{\sin(\theta)}{\hat{r}}, \tag{4.8}$$

The inner potential behaves like a two-dimensional vertical dipole whose intensity $\mu(\tilde{x})$ is equal to the three-dimensional dipole intensity of the outer solution. Matching the free surface elevations at infinity enables one to conclude that the inner free surface elevation decreases as the inverse square of the distance:

$$\lim_{\hat{y}\to\infty}\hat{\eta}_1(\tilde{x},\hat{y}) = \frac{1}{\hat{y}^2}\int_0^{\tilde{x}} -\frac{\mu(s)}{2\pi}\,\mathrm{d}s \tag{4.9}$$

Since the outer solution remains valid in the inner domain in front of the ship ($\tilde{x} < 0$), the initial conditions of the inner problem are found by matching directly the inner solution to the outer one. This matching condition, which is discussed in

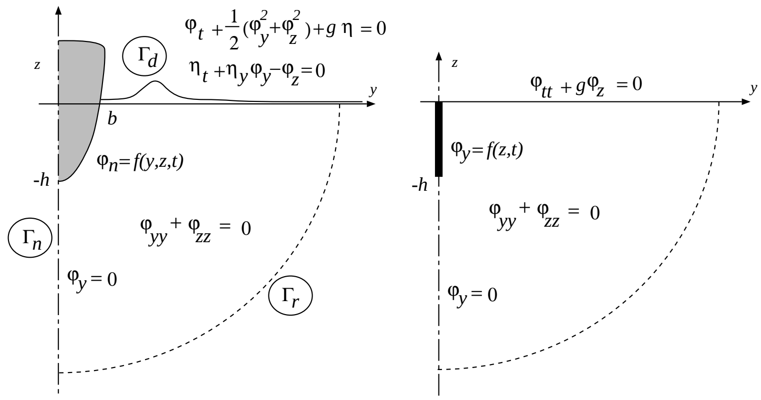


Figure 3. Formulation of the inners problems in a reference framed fixed with respect to the fluid: slender ship (left), thin ship (right)

more details in Fontaine (1996), leads to:

$$\hat{\eta}_1(\tilde{x} = 0, \hat{y}) = 0, \quad \hat{\varphi}_1(\tilde{x} = 0, \hat{y}, \hat{z} = 0) = 0. \tag{4.10}$$

As a consequence, in front of the ship, the fluid is at rest and the free surface is not perturbed. At leading order, the outer solution does not influence the inner one. The only interaction that appears between the strips is due to the free surface conditions.

## 5. Numerical solution of the inner problem

Consider an observer fixed in a tranverse plane, perpendicular to the velocity $\boldsymbol{U}$ and translating with the uniform stream, see figure 3. For this observer, the two-dimensional inner flow appears to be generated by the change of area of the section of the ship that crosses the tranverse plane. Thus, the ship can be considered as a two-dimensional wave maker that generates nonlinear waves. The variable $\tilde{x}$ corresponds to time and this theory is classically called '2D+t'.

To solve numerically the inner problem, we use a Mixed Eulerian–Lagrangian method (MEL) similar to that introduced by Longuet-Higgins & Cokelet (1976) based on the Sindbad code (Cointe 1990).

The free surface boundary conditions (2.7)–(2.8) are reformulated using a Lagrangian description. They are written for fluid particles on the free surface ($\boldsymbol{X} = (y, z) \in \Gamma_d(t)$) and the associated potential $\varphi(\boldsymbol{X})$, yielding

$$\frac{D\varphi}{Dt} = -gz + \frac{1}{2}\left[\left(\frac{\partial\varphi}{\partial s}\right)^2 + \left(\frac{\partial\varphi}{\partial n}\right)^2\right], \tag{5.1}$$

$$\frac{D\boldsymbol{X}}{Dt} = \frac{\partial\varphi}{\partial s}\boldsymbol{s} + \frac{\partial\varphi}{\partial n}\boldsymbol{n}, \tag{5.2}$$

where $D$ is used to indicate a material derivative and $\boldsymbol{s}$ and $\boldsymbol{n}$ are vectors tangent and normal to the free surface, respectively.

A Neumann boundary condition is imposed on the hull as a result of the body boundary condition (2.6) as well as on the axes of symmetry below the ship.

Far from the ship, the fluid domain is bounded by a control surface $\Gamma_r$ on which is applied the matching condition with the outer solution. For a circular surface control, equation (4.8) leads to a Robin–Fourier condition:

$$\varphi + r\frac{\partial \varphi}{\partial n} = 0. \tag{5.3}$$

The kinematic constraint $\Delta\varphi = 0$ with the boundary condition on $\Gamma_n$ and $\Gamma_r$ permits the free surface boundary conditions (5.1) and (5.2) to be expressed as an evolution equation for $(\varphi, \boldsymbol{X})$. This stems from the fact that, if at a given time $t$, $\varphi$ is known along $\Gamma_d$ and $\varphi_n$ is known along $\Gamma_n$, then $\varphi_n$ can be computed along the free surface and the right-hand side of (5.1)–(5.2) can be evaluated. For that purpose, we use the integral equation

$$\theta(M)\varphi(M) = \int_{\Sigma}\left[\varphi(P)\frac{\partial G}{\partial n_P}(M,P) - \frac{\partial \varphi}{\partial n_P}(P)G(M,P)\right]\,\mathrm{d}s_P, \tag{5.4}$$

where $M$ is a point on the boundary, $G$ is the simple source Green function, $\theta(M)$ is the included angle, or the angle between two tangents of the boundary at $M$ (equal to $\pi$ for a smooth curve) and $s$ a curvilinear abscissa along $\Sigma = \Gamma_n + \Gamma_d + \Gamma_r$. Equation (5.4) is discretized using a standard collocation method. The boundary of the domain is approximated by segments and $\varphi$ and $\varphi_n$ are assumed to vary linearly along each segment. This allows an analytical integration of the Green's function, its normal derivative and their products along the curvilinear abscissa so that the calculation is rather simple and vectorizes well. The resulting linear system is solved using an iterative method (GMRes) starting from the solution at the preceeding time step.

The resulting evolution equations are solved numerically using standard time-stepping procedures, such as a fourth-order Runge–Kutta algorithm.

## 6. Applications

### (*a*) *Flow around a wedge shaped bow*

Experiments have been carried out for a wedge shaped bow (Fontaine *et al.* 1995). The comparison with theory shows that if the linear thin ship theory (Ogilvie 1972) gives adequate predictions of the characteristic length scales of the wave pattern, it underpredicts the bow wave amplitude. The present numerical computations based on the nonlinear slender ship theory are in much better agreement with experiments (see figure 4) except at the nose where an initial wave elevation is measured. The failure of the linear thin ship theory is due to the fact that a wedge shaped bow can never be assumed to be thin because the theoretical fluid acceleration is not finite at its 'nose' (Fontaine 1996).

### (*b*) *Flow around a prismatic planing hull*

For such a flow, there is no length scale characteristic of the geometry of the hull. In a domain near the nose, i.e. for $gx/U^2 \ll 1$, gravity can be neglected and dimensional analysis implies that the flow is self-similar. The planing problem is then comparable to the two-dimensional gravity-free impact problem.

It can been shown (Fontaine 1996), using the formalism of matched asymptotic expansions, that the first order composite solution describing the flow during the water entry of a wedge with small deadrise angle (e.g. Cointe & Armand 1987; Howison *et al.* 1991; Cointe 1991) is also a solution of the inner problem for the flow around a flat prismatic hull (see figure 5). This justifies the pioneering results of

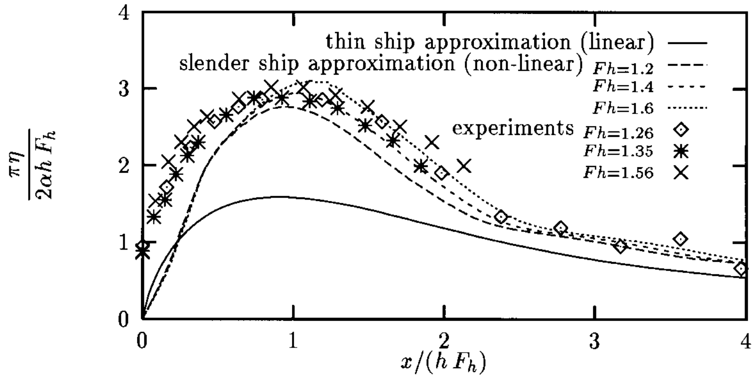


Figure 4. Non-dimensionnal wave profile along the wedge shaped bow.

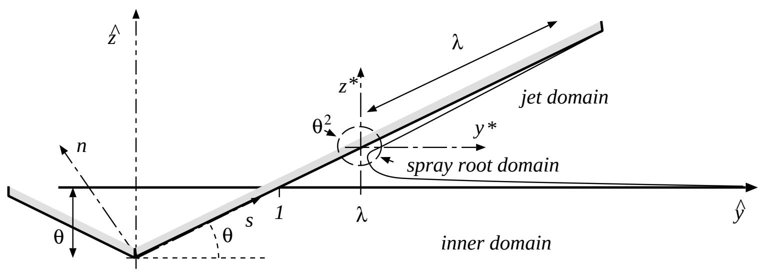


Figure 5. Tranverse cut of the flow around a flat prismatic hull.

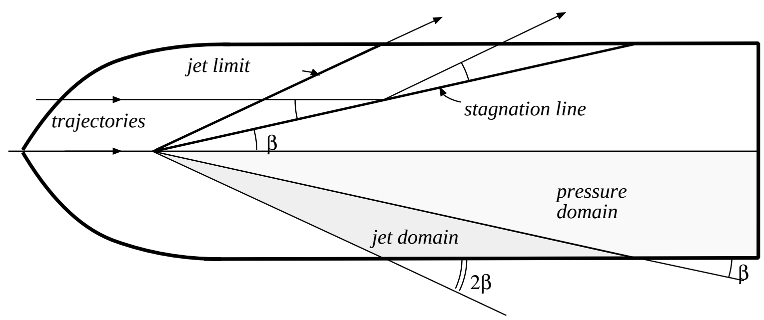


Figure 6. Stucture of the flow around a flat V shaped planing hull.

Wagner (1932). Moreover, it allows the flow near the planing hull to be determined, see figure 6. Far from the prismatic hull, the three-dimensional flow is given by the outer solution defined in section 4.

Near the hull, the two-dimensional inner solution corresponds to the flow around a flat plate normal to the upstream velocity and of width equal to the wetted width of the hull section. This flow is singular at the extremities of the plate and the inner solution is matched to a nonlinear solution describing the spray root. This nonlinear solution involves a jet with constant thickness and velocity. This latter solution is matched to another one describing the development of the jet along the hull. At leading order, the thickness of the jet is linearly decreasing along the hull, the fluid velocity in the jet is constant, and the length of the jet is equal to half the wetted width. The angle between the jet limit and the stagnation line is then equal to the angle between the stagnation line and the symmetry axis of the ship.

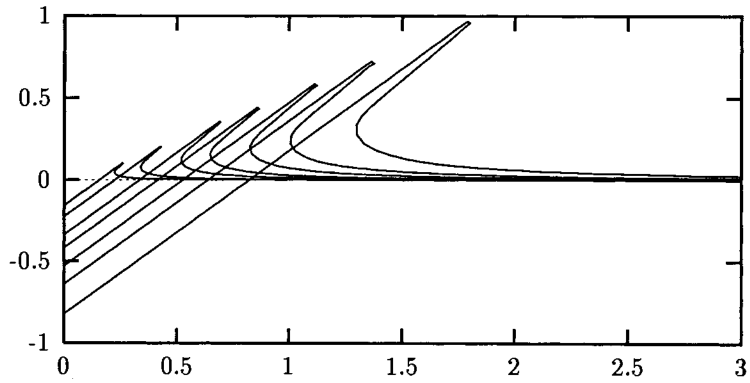


Figure 7. Evolution of the free surface elevation near a prismatic hull ($\theta = 45°$).

The analytical solution describing the jet has been used to derive a numerical treatment used to model the jet. During the simulation, the upper part of the jet is truncated. A new segment perpendicular to the hull is introduced to bound the fluid domain. On this segment, the fluid normal velocity is constant and equal to the tangential fluid velocity along the hull. This numerical treatment enables one to follow the development of the jet, see figure 7. The resulting numerical solution is in good agreement with the self-similar solution of Zhao & Faltinsen (1993). In particular, the intersection angle between the jet and the hull is correctly predicted.

## References

Chapman, R. B. 1975 Free surface effects for yawed surface-piercing plate. *J. Ship Res.* **20**, 125–136.

Cointe, R. 1990 Numerical simulation of a wavetank. *Engng Anal. Boundary Elements* **7**, 167–177.

Cointe, R. 1991 Free-surface flows close to a surface-piercing body. In *Mathematical approaches in Hydrodynamics* (ed. T. Miloh), pp. 319–334. Philadelphia: SIAM.

Cointe, R. & Armand, J.-L. 1987 Hydrodynamic impact analysis of a cylinder. *ASME J. Offshore Mech. Arctic Engng* **109**, 237–243.

Fontaine, E. 1996 Simulation de l'écoulement potentiel engendré par un corps élancé perçant la surface libre à forts nombres de Froude., Ph.D. thesis., Paris: Ecole Nationale des Ponts et Chaussées.

Fontaine, E., Cointe, R. & Rivoalen, E. 1995 Etude de la vague d'étrave d'un navire élancé. In *Proc. cinquièmes journées de l'hydrodynamique* Rueil-Malmaison: IFP.

Faltinsen, O. M. & Zhao, R. 1991 Flow prediction around high-speed ships in waves. In *Mathematical approaches in hydrodynamics* (ed. T. Miloh), pp. 265–288. Philadelphia: SIAM.

Howison, S. D., Ockendon, J. R., & Wilson, S. K. 1991 Incompressible water entry problems at small deadrise angles. *J. Fluid Mech.* **222**, 215–230.

Kaplun, S. & Lagerstrom, P. A. 1957 Asymptotic expansions of Navier–Stokes solutions for small Reynolds numbers. *J. Math. Mech.* **6**, 585–593.

Longuet-Higgins, M. S. & Cokelet, E. D. 1976 The deformation of steep surface waves on water. I. A numerical method of computation. *Proc. R. Soc. Lond.* A **350**, 1–26.

Maruo, H. & Song, W. 1994 Nonlinear analysis of bow wave breaking and deck wetness of a high-speed ship by the parabolic approximation. In *Proc. 20th Symp. on Naval Hydrodynamics*. Washington, DC: National Academy Press.

Ogilvie, T. F. 1967 Nonlinear high-Froude-number free-surface problems. *J. Engng Math.* **1**, 215–235.

Ogilvie, T. F. 1972 The wave generated by a fine ship bow. In *Proc. 9th Symp. Naval Hydrodynamics*, vol. 2, pp. 1483–1525. Washington, DC: National Academy Press.

Tulin, M. P. 1956 The theory of slender surfaces planing at high speed. *Shiffstechnik* **4**, 125–133.

Wagner, H. 1932 Über Sto$\beta$ und Gleitvorgänge an der Oberfläche von Flüssigkeiten. *ZAMM* **12**, 193–215.

Ward, G. N. 1955 *Linearised theory of steady high-speed flow.* Cambridge University Press.

Zhao, R. & Faltinsen, O. 1993 Water entry of two-dimensional bodies. *J. Fluid Mech.* **246**, 593–612.